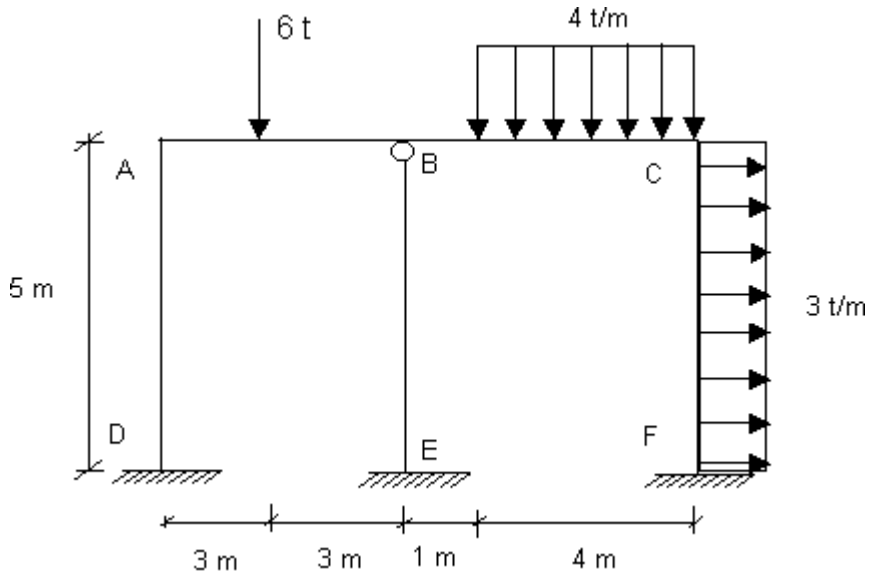
Hallar por el método de Cross los diagramas de momento flector y de esfuerzo cortante, así como las reacciones de la estructura de la figura, indicando la situación de los puntos más característicos de ambos diagramas utilizando el método de superposición.

Determinar el ángulo girado por C en la barra BC.

Los momentos de inercia de las barras horizontales es I_1, mientras que el de las barras verticales es I_2, sabiendo que la relación entre ellos es $I_1=2 \times I_2$.



1º Coeficientes elásticos

NUDO A

$$K_{AD} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_2}{5} = \frac{2 \cdot E \cdot I_1}{5} \qquad \beta_{BA} = 1/2$$

$$K_{AB} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{6} = \frac{2 \cdot E \cdot I_1}{3} \qquad \beta_{AB} = 1/2$$

$$r_{AD} = \frac{2/5}{2/5 + 2/3} = 0.37$$

$$r_{AB} = \frac{2/3}{2/5 + 2/3} = 0.63$$

NUDO B

$$K_{BA} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{6} = \frac{2}{3} \cdot E \cdot I_1 \qquad \beta_{BA} = 1/2$$

$$K_{BC} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{5} \qquad \beta_{BC} = 1/2$$

$$K_{BE} = 0 \qquad \beta_{BE} = 1$$

$$r_{BA} = \frac{2/3}{2/3 + 4/5 + 0} = 0.45$$

$$r_{BC} = \frac{4/5}{2/3 + 4/5 + 0} = 0.55$$

$$r_{BE} = 0$$

NUDO C

$$K_{CB} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{5} \qquad \beta_{CB} = 1/2$$

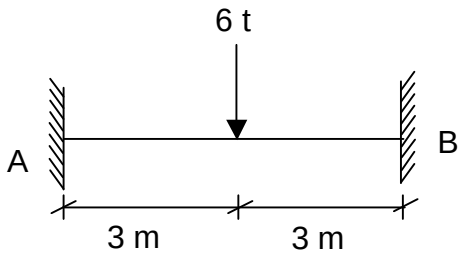
$$K_{CF} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_2}{5} = \frac{2 \cdot E \cdot I_1}{5} \qquad \beta_{CF} = 1/2$$

$$r_{CB} = \frac{4/5}{4/5 + 2/5} = 0.67$$

$$r_{CF} = \frac{2/5}{4/5 + 2/5} = 0.33$$

2º. Calculamos los momentos y pares de empotramiento

Tramo AB

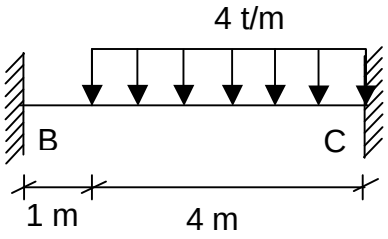


$$M_A = M_B = -\frac{P \cdot l}{8} = -\frac{6 \cdot 6}{8} = -4.5\ t \cdot m$$

$$m_A = +4.5\ t \cdot m$$

$$m_B = -4.5\ t \cdot m$$

Tramo BC



a =3 m
b =2 m
c =4 m
q =4 t/m
l =5 m

$$M_B = \frac{-q \cdot c^3}{12 \cdot l^2} \cdot \left(l - 3 \cdot b + \frac{12 \cdot a \cdot b^2}{c^2} \right)$$

$$M_B = \frac{-4 \cdot 4^3}{12 \cdot 5^2} \cdot \left(5 - 3 \cdot 2 + \frac{12 \cdot 3 \cdot 2^2}{4^2} \right) = -6.83\ t \cdot m$$

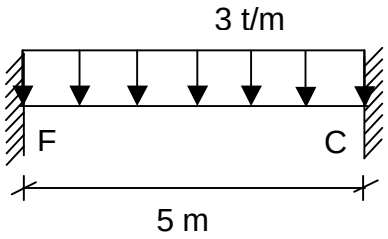
$$M_C = \frac{-q \cdot c^3}{12 \cdot l^2} \cdot \left(l - 3 \cdot a + \frac{12 \cdot a^2 \cdot b}{c^2} \right)$$

$$M_C = \frac{-4 \cdot 4^3}{12 \cdot 5^2} \cdot \left(5 - 3 \cdot 3 + \frac{12 \cdot 3^2 \cdot 2}{4^2} \right) = -8.11\ t \cdot m$$

$$m_B = +6.83\ t \cdot m$$

$$m_C = -8.11\ t \cdot m$$

Tramo FC

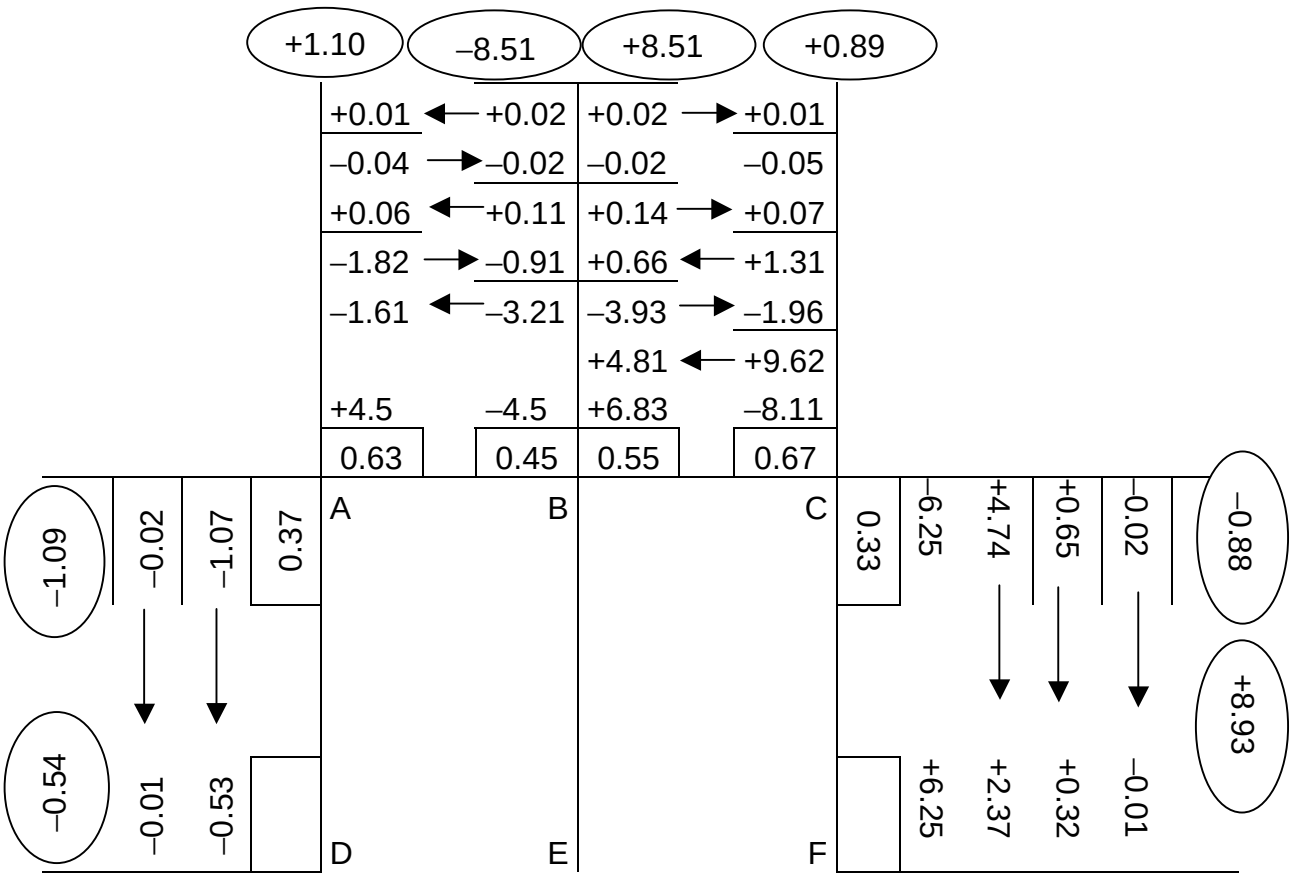


$$M_F = M_C = -\frac{q \cdot l^2}{12} = -\frac{3 \cdot 5^2}{12} = -6.25\ t \cdot m$$

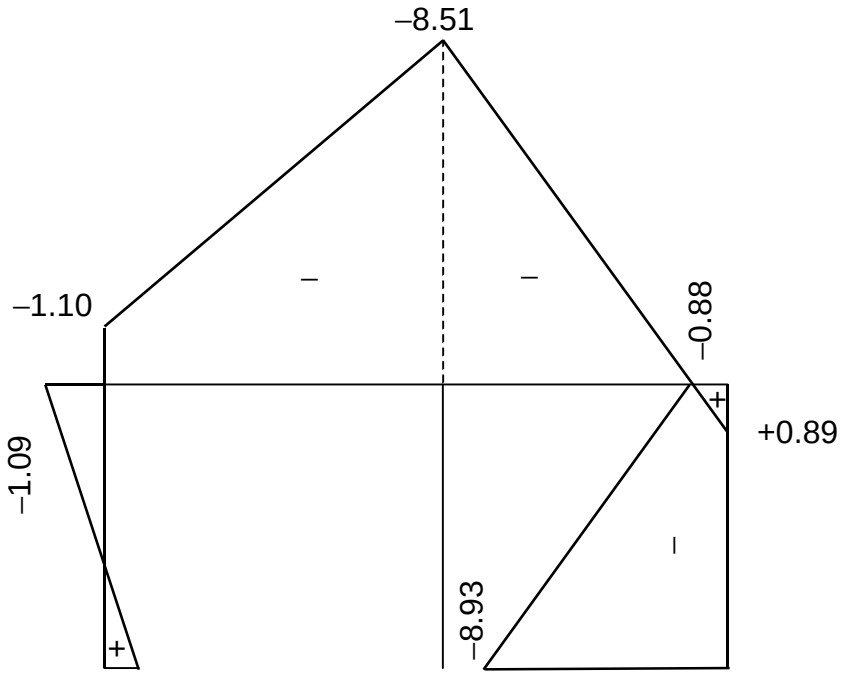
$$m_F = +6.25\ t \cdot m$$

$$m_C = -6.25\ t \cdot m$$

3º Cross: Transmisiones

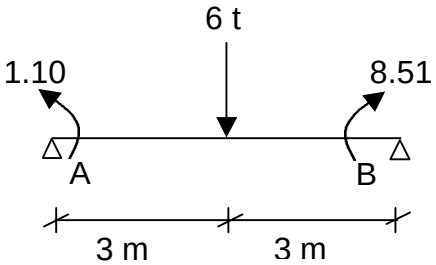


4º Diagrama de momentos

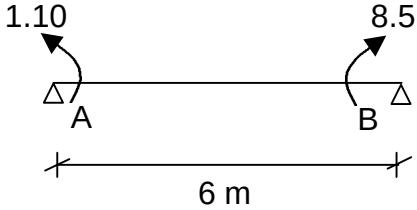


Obtención de los momentos máximos en las barras cargadas

Tramo AB

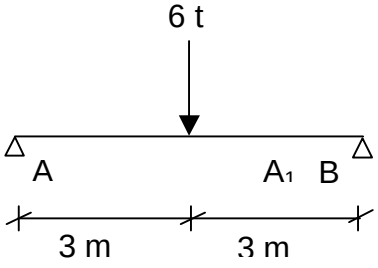


[1]



+

[2]



[1]

$$M_{AB} = -\frac{M_a}{l}\cdot(l-x)-\frac{M_b}{l}\cdot x$$

$$M_{AB} = -\frac{1.10}{6}\cdot(6-x)-\frac{8.51}{6}\cdot x = -1.10-1.235\cdot x$$

[2]

$$M_{AA_1} = \frac{P}{2}\cdot x = 3\cdot x$$

$$M_{A_1B} = \frac{P}{2}\cdot(l-x) = 3\cdot(6-x) = 18-3\cdot x$$

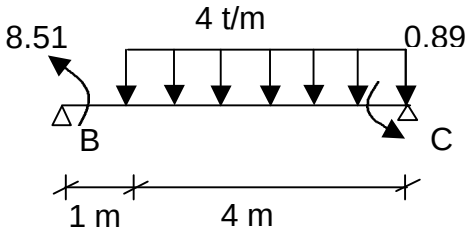
Superponiendo:

$$M_{AB} + M_{AA_1} = -1.10-1.235\cdot x+3\cdot x = -1.10+1.765\cdot x$$

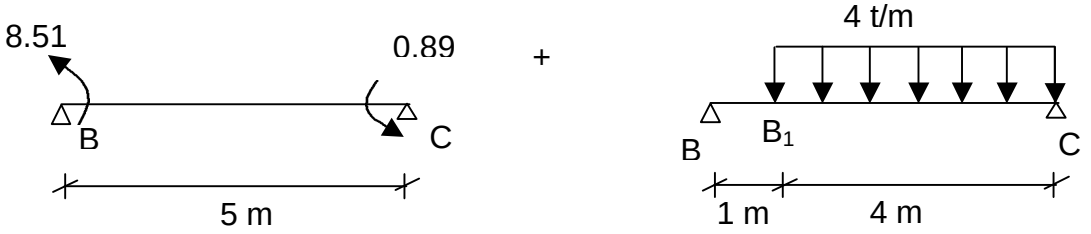
$$M_{AB} + M_{A_1B} = -1.10-1.235\cdot x+48-3\cdot x = 16.9-4.235\cdot x$$

En x =3, $M_{x=3} = -1.10+1.765\cdot 3 = 4.195\ t\cdot m$

Tramo BC



[1] [2]



[1]

$$M_{BC} = -\frac{M_b}{l}\cdot(l-x)-\frac{M_c}{l}\cdot x$$

$$M_{BC} = -\frac{8.51}{5}\cdot(5-x)-\frac{0.88}{5}\cdot x = -8.51+1.878\cdot x$$

[2]

$$M_{BB1} = \frac{q\cdot b\cdot c}{l}\cdot x = \frac{4\cdot 2\cdot 4}{5} = 6.4\cdot x$$

$$M_{B_1C} = \frac{q\cdot b\cdot c}{l}\cdot x - \frac{q}{2}\cdot\left(x-\left(a-\frac{c}{2}\right)\right)^2$$

$$M_{B_1C} = \frac{4\cdot 2\cdot 4}{5}\cdot x - \frac{4}{2}\cdot\left(x-\left(3-\frac{4}{2}\right)\right)^2 = 6.4\cdot x - 2\cdot(x-1)^2 = -2\cdot x^2+10.4\cdot x-4$$

Superponiendo:

$$M_{BC}+M_{BB_1} = -8.51+1.878\cdot x+6.4\cdot x = -8.51+8.278\cdot x$$

$$M_{BC}+M_{B_1C} = -8.51+1.878\cdot x-2\cdot x^2+10.4\cdot x-4 = -2\cdot x^2+12.278\cdot x-12.51$$

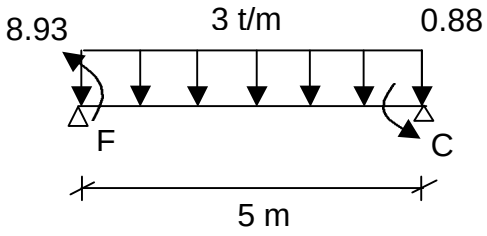
M' =0 $\qquad -4\cdot x+12.278=0$

$$x = 12.278/4 = 3.07\text{m}$$

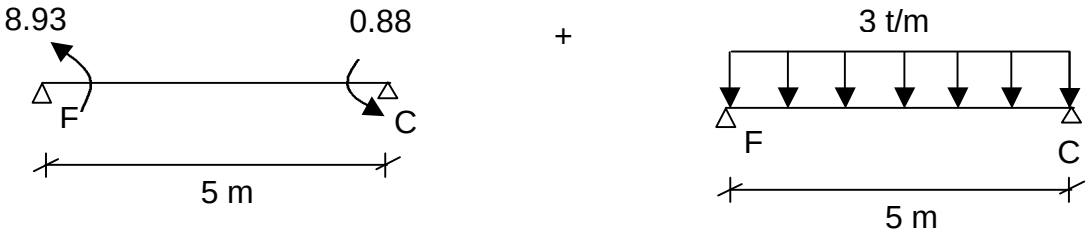
M''< 0 $\rightarrow$ máximo

$$M_{(x=3.07)} = -2\cdot 3.07^2+12.278\cdot 3.07-12.51 = 6.33\text{t}\cdot\text{m}$$

Tramo FC



[1] [2]



[1]

$$M_{FC} = -\frac{M_f}{l}\cdot(l-x)+\frac{M_c}{l}\cdot x$$

$$M_{FC} = -\frac{8.93}{5}\cdot(5-x)-\frac{0.88}{5}\cdot x = -8.93+1.61\cdot x$$

[2]

$$M_{FC} = \frac{q\cdot x}{2}\cdot(l-x) = \frac{3\cdot x}{2}\cdot(5-x) = 7.5-1.5\cdot x^2$$

Superponiendo:

$$M_{FC} = -8.93+1.61\cdot x+7.5\cdot x-1.5\cdot x^2 = -1.5\cdot x^2+9.11\cdot x-8.93$$

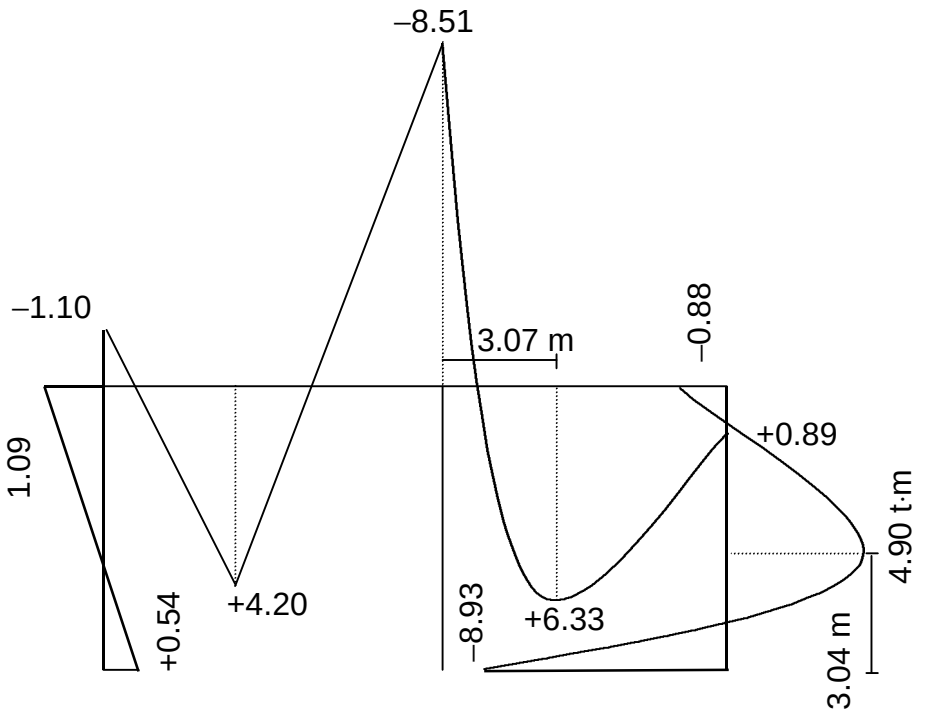
M' =0 $\quad -3\cdot x+9.11=0$

$x = 3.04\,m$

M''< 0 $\rightarrow$ máximo

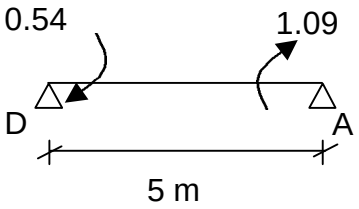
$$M_{(x=3.04)} = -1.5\cdot 3.04^2+9.11\cdot 3.04-8.93 = 4.90\,t\cdot m$$

4º. Diagrama de momentos flectores



5º Reacciones y diagrama de esfuerzo cortante

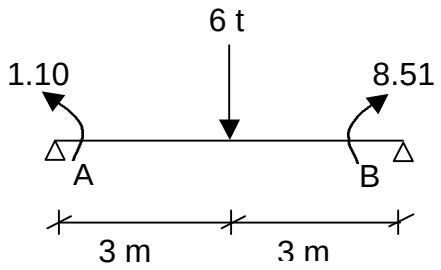
Tramo DA



$$\sum M_A = 0$$
$$R_D \cdot 5 + 0.54 + 1.09 = 0$$
$$R_D = -0.33\ t$$

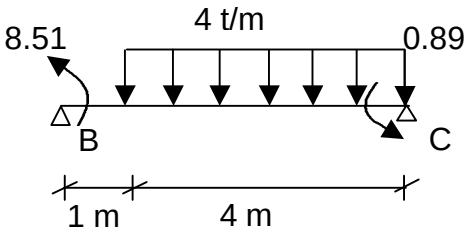
$$\sum M_D = 0$$
$$R_C \cdot 5 - 0.54 - 1.09 = 0$$
$$R_C = +0.33\ t$$

Tramo AB



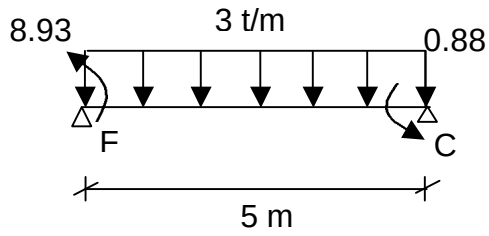
$$\sum M_A = 0$$
$$R_B \cdot 6 - 6 \cdot 3 - 8.51 + 1.10 = 0$$
$$R_B = 4.23 \text{ t}$$
$$\sum M_B = 0$$
$$R_A \cdot 6 - 6 \cdot 3 + 8.51 - 1.10 = 0$$
$$R_A = 1.77 \text{ t}$$

Tramo BC

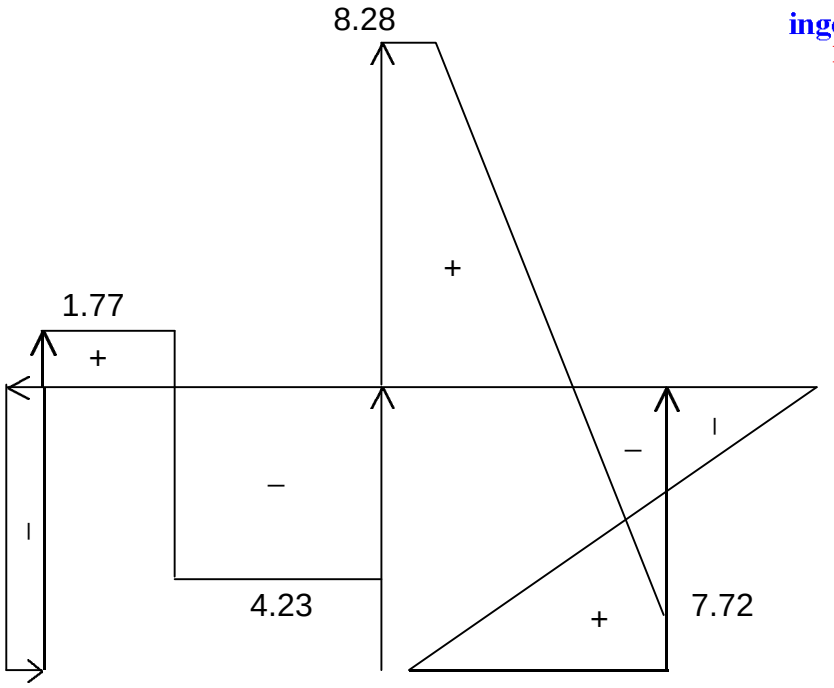


$$\sum M_C = 0$$
$$R_B \cdot 5 - 4 \cdot 4 \cdot 2 - 8.51 - 0.89 = 0$$
$$R_B = 8.28 \text{ t}$$
$$\sum M_B = 0$$
$$R_C \cdot 5 - 4 \cdot 4 \cdot 3 + 8.51 + 0.89 = 0$$
$$R_A = 7.72 \text{ t}$$

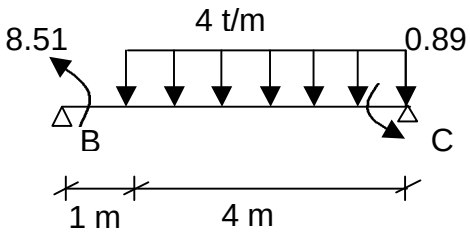
Tramo FC

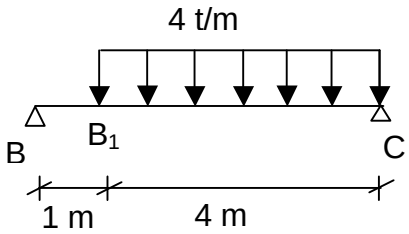


$$\sum M_C = 0$$
$$R_F \cdot 5 - 5 \cdot 3 \cdot 2.5 - 8.93 + 0.88 = 0$$
$$R_F = 9.11 \text{ t}$$
$$\sum M_F = 0$$
$$R_C \cdot 5 - 5 \cdot 3 \cdot 2.5 + 8.93 - 0.88 = 0$$
$$R_C = 5.89 \text{ t}$$

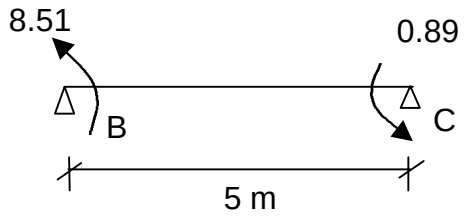


6º Angulo girado en C por la barra BC





$$\varphi_C = -\frac{q \cdot b \cdot a \cdot c}{6 \cdot E \cdot I \cdot l} \cdot \left(l + a - \frac{c^2}{4 \cdot b} \right)$$



$$\varphi_C = \frac{l}{6 \cdot E \cdot I} \cdot (M_b + 2 \cdot M_c)$$

$$\varphi_C = -\frac{5}{6 \cdot E \cdot I} \cdot (2 \cdot 0.89 - 8.51) - \frac{4 \cdot 2 \cdot 4 \cdot 3}{6 \cdot E \cdot I \cdot 5} \cdot \left(5 + 3 - \frac{4^2}{4 \cdot 2} \right) = -\frac{13.59}{E \cdot I} \text{ rad ; (E, I en t y m)}$$