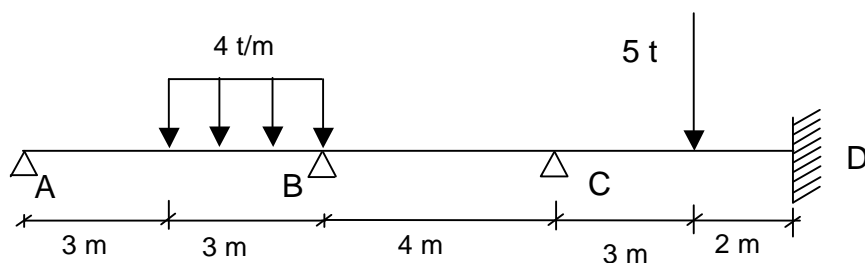


Hallar por el método de Cross los diagramas de momento flector y esfuerzo cortante, así como las reacciones de la viga de la figura, indicando la situación de los puntos más característicos de ambos diagramas utilizando el método de superposición.

Determinar el ángulo girado por B en la barra AB.



1º Determinamos los coeficientes elásticos.

NUDO B

$$K_{BA} = \frac{3 \cdot E \cdot I}{6} = 0.5 \cdot E \cdot I$$

$$\beta_{BA} = 0$$

$$K_{BC} = \frac{4 \cdot E \cdot I}{4} = E \cdot I$$

$$\beta_{BC} = \frac{1}{2}$$

$$r_{BA} = \frac{0.5}{0.5 + 1} = 0.33$$

$$r_{BC} = \frac{1}{0.5 + 1} = 0.67$$

NUDO C

$$K_{CB} = \frac{4 \cdot E \cdot I}{4} = E \cdot I$$

$$\beta_{CB} = \frac{1}{2}$$

$$K_{CD} = \frac{4 \cdot E \cdot I}{5} = 0.8 \cdot E \cdot I$$

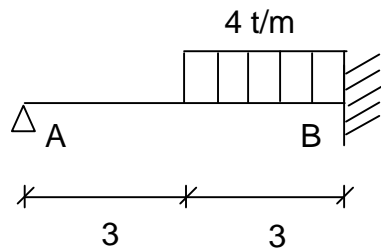
$$\beta_{CD} = \frac{1}{2}$$

$$r_{CB} = \frac{1}{1 + 0.8} = 0.56$$

$$r_{CD} = \frac{0.8}{1 + 0.8} = 0.44$$

2º. Calculamos los momentos y pares de empotramiento

Tramo AB



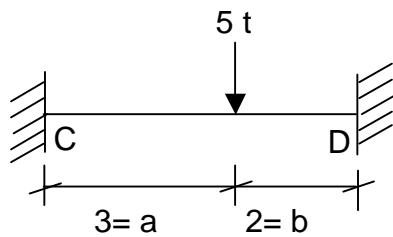
$$M_A = 0$$

$$M_B = -\frac{q \cdot a \cdot b \cdot c}{2 \cdot l^2} \cdot \left(l + a - \frac{c^2}{4 \cdot b} \right)$$

$$a = 4.5 ; b = 1.5 ; c = 3 ; l = 6 ; q = 4$$

$$M_B = -\frac{4 \cdot 4.5 \cdot 1.5 \cdot 3}{2 \cdot 6^2} \cdot \left(6 + 4.5 - \frac{3^2}{4 \cdot 1.5} \right) = -10.125 \text{ t} \cdot \text{m}$$

Tramo CD



$$M_C = -\frac{P \cdot a \cdot b^2}{l^2} = -\frac{5 \cdot 3 \cdot 2^2}{5^2} = -2.4 \text{ t} \cdot \text{m}$$

$$M_D = -\frac{P \cdot a^2 \cdot b}{l^2} = -\frac{5 \cdot 3^2 \cdot 2}{5^2} = -3.6 \text{ t} \cdot \text{m}$$

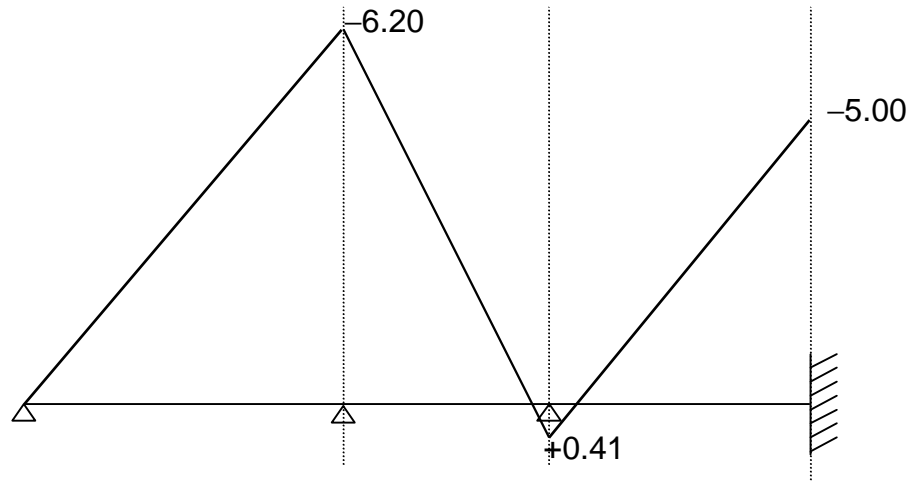
Pares de empotramiento

$$m_A = 0 \text{ t} \cdot \text{m} \quad m_B = -10.13 \text{ t} \cdot \text{m} \quad m_C = +2.4 \text{ t} \cdot \text{m} \quad m_D = -3.6 \text{ t} \cdot \text{m}$$

3º. Cross: Transmisiones

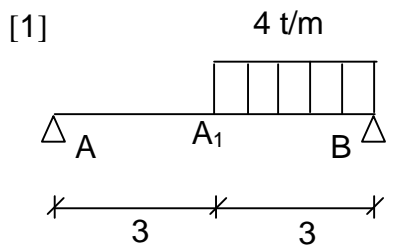
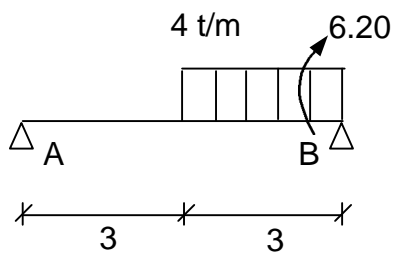
0	-6.21	+6.20	+0.41	-0.41	-5.00
	-0.01	-0.03	-0.02	→ -0.01	
+0.05	+0.10	→ +0.05			
-0.15	← -0.31	-0.24	→ -0.12		
+0.53	+1.09	→ +0.55			
-1.62	← -3.24	-2.55	→ -1.27		
+3.34	+6.79	→ +3.39			
-10.13			+2.40	-3.60	
	0.33	0.67	0.56	0.44	
A	B	C	D		

4º. Diagrama de momentos flectores en los nudos

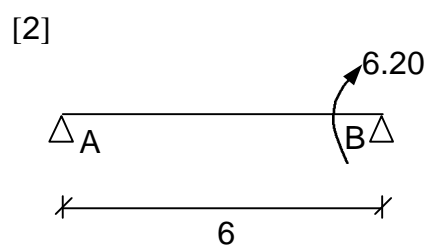


Momentos isostáticos:

Tramo AB



+

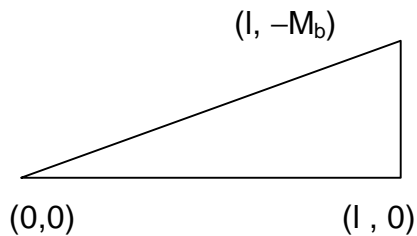


$$[1] \quad M_{AA_1} = \frac{q \cdot b \cdot c}{l} \cdot x = \frac{4 \cdot 1.5 \cdot 3}{6} = 3 \cdot x$$

$$M_{A_1B} = \frac{q \cdot b \cdot c}{l} \cdot x - \frac{q}{2} \cdot \left(x - \left(a - \frac{c}{2} \right) \right)^2$$

$$M_{A_1B} = \frac{4 \cdot 1.5 \cdot 3}{6} \cdot x - \frac{4}{2} \cdot \left(x - \left(4.5 - \frac{3}{2} \right) \right)^2 = 3 \cdot x - 2 \cdot (x - 3)^2 = -2 \cdot x^2 + 15 \cdot x - 18$$

$$[2] \quad M_{AB} = -M \cdot \frac{x}{l} = -\frac{6.20}{6} \cdot x = -1.03 \cdot x$$



$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} \quad \frac{x-0}{l-0} = \frac{M-0}{-M_B-0}$$

$$M = \frac{-M_B \cdot x}{l}$$

Superponiendo:

$$M_{AA_1} = 3 \cdot x - 1.03 \cdot x = -1.97 \cdot x \rightarrow \text{En } x=3 \quad M = +5.91 \text{ t}\cdot\text{m}$$

$$M_{A_1B} = -2x^2 + 15 \cdot x - 18 - 1.03 \cdot x = -2 \cdot x^2 + 13.97 \cdot x - 18$$

Puntos de corte (o puntos de momento nulo):

$$-2 \cdot x^2 + 13.97 \cdot x - 18 = 0$$

$$+2 \cdot x^2 - 13.97 \cdot x + 18 = 0$$

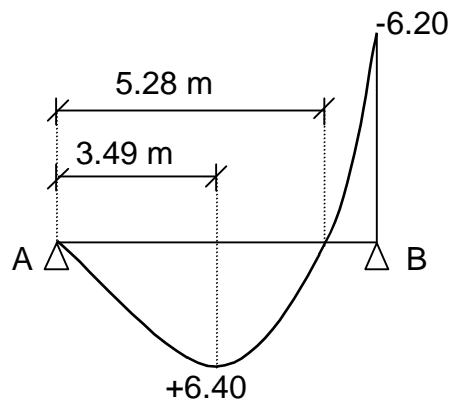
$$x = \begin{cases} 1.70 \text{ m} \rightarrow \notin A_1B \\ 5.28 \text{ m} \rightarrow \text{Punto de corte con el eje } x \end{cases}$$

Máximos y mínimos:

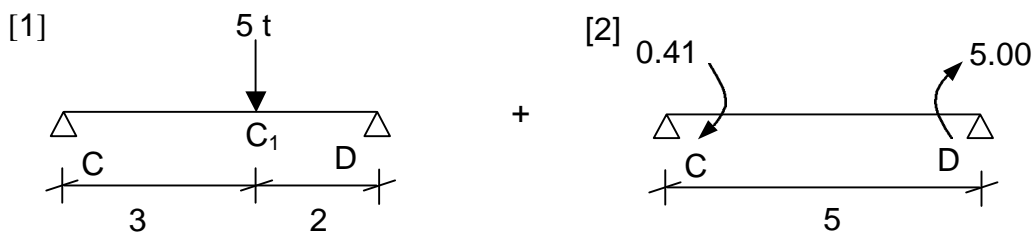
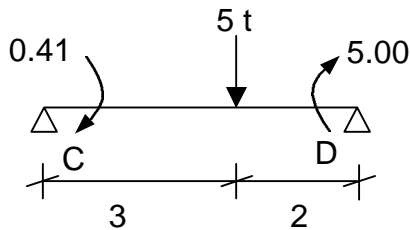
$$-4 \cdot x + 13.97 = 0 \rightarrow x = 0.49 \text{ m}$$

$$-4 < 0 \text{ (máximo)}$$

$$M_{x=3.49} = -2 \cdot 3.49^2 + 13.97 \cdot 3.49 - 18 = 6.40 \text{ t}\cdot\text{m}$$



Tramo CD



$$[1] \quad M_{CC_1} = \frac{P \cdot b}{l} \cdot x = \frac{5 \cdot 2}{5} = 2 \cdot x$$

$$M_{C_1D} = \frac{P \cdot a}{l} \cdot (l - x) = \frac{5 \cdot 2}{5} \cdot (5 - x) = 15 - 3 \cdot x$$

$$[2] \quad M_{CD} = \frac{M_a}{l} \cdot (l - x) - \frac{M_b}{l} \cdot x = \frac{0.41}{5} \cdot (5 - x) - \frac{5.00}{5} \cdot x$$

$$M_{CD} = 0.41 - 0.08 \cdot x - 1.00 \cdot x = 0.41 - 1.08 \cdot x$$

Superponiendo:

$$M_{CC_1} = 2 \cdot x + 0.41 - 1.08 \cdot x = 0.92 \cdot x + 0.41$$

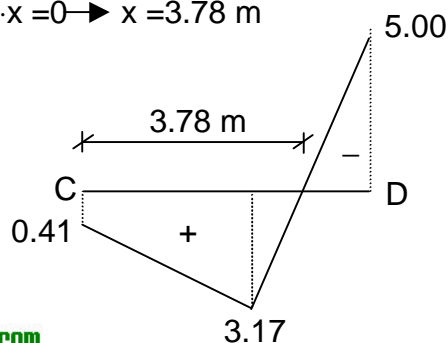
$$M_C = 0.41$$

$$M_{C_1} = 0.92 \cdot 3 + 0.41 = +3.17 \text{ t}\cdot\text{m}$$

$$M_{CC_1} = 0.41 - 1.08 \cdot x + 15 - 3 \cdot x = 15.41 - 4.08 \cdot x$$

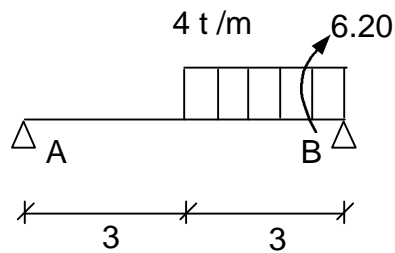
$$\text{En } x = 3 \quad M_{C_1} = +3.17 \text{ t}\cdot\text{m} \quad \text{En } x = 5 \quad M_D = -4.99 \text{ t}\cdot\text{m}$$

$$15.41 - 4.08 \cdot x = 0 \rightarrow x = 3.78 \text{ m}$$



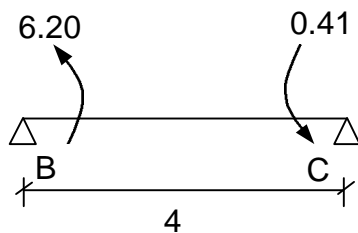
5º. Reacciones y esfuerzo cortante:

Tramo AB



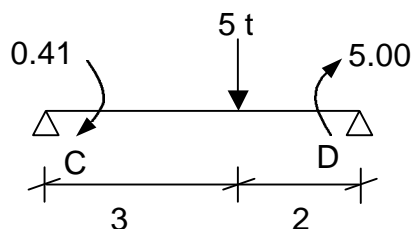
$$\begin{aligned}\sum M_A &= 0 \\ R_B \cdot 6 - 6.20 - 4 \cdot 3 \cdot 4.5 &= 0 \\ R_B &= 10.03 \text{ t} \\ \sum M_B &= 0 \\ R_A \cdot 6 + 6.20 - 4 \cdot 3 \cdot 1.5 &= 0 \\ R_A &= 1.97 \text{ t}\end{aligned}$$

Tramo BC



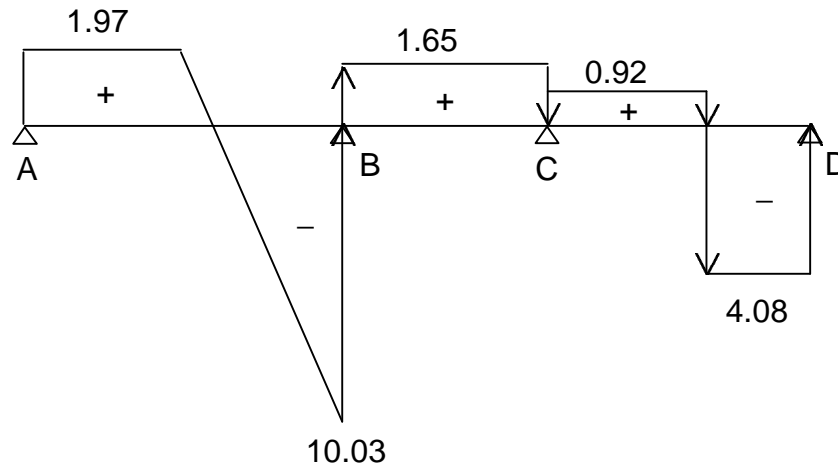
$$\begin{aligned}\sum M_B &= 0 \\ R_C \cdot 4 + 6.20 + 0.41 &= 0 \\ R_C &= -1.65 \text{ t} \\ \sum M_C &= 0 \\ R_B \cdot 4 - 6.20 - 0.41 &= 0 \\ R_B &= +1.65 \text{ t}\end{aligned}$$

Tramo CD

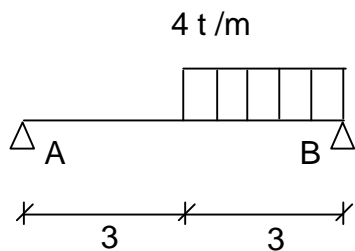
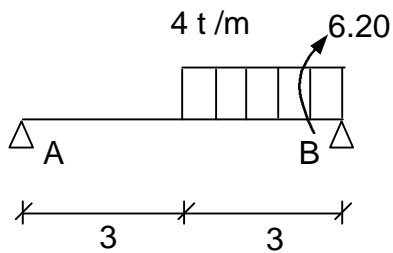


$$\begin{aligned}\sum M_C &= 0 \\ R_D \cdot 5 - 5.00 - 0.41 - 5 \cdot 3 &= 0 \\ R_D &= 4.08 \text{ t} \\ \sum M_D &= 0 \\ R_C \cdot 5 + 5.00 - 0.41 - 5 \cdot 2 &= 0 \\ R_C &= 0.92 \text{ t}\end{aligned}$$

Diagrama de esfuerzos cortantes

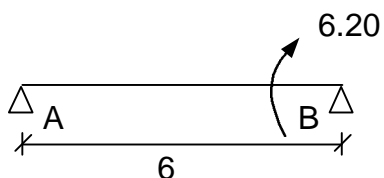


6º. Angulo girado en B



$$\varphi_B = -\frac{q \cdot a \cdot b \cdot c}{6 \cdot E \cdot I \cdot l} \left(l + a - \frac{c^2}{4 \cdot b} \right)$$

$$\varphi_B = -\frac{4 \cdot 4.5 \cdot 1.5 \cdot 3}{6 \cdot E \cdot I \cdot 6} \left(6 + 4.5 - \frac{3^2}{4 \cdot 1.5} \right) = -\frac{20.25}{E \cdot I}$$



$$\varphi_B = \frac{M \cdot l}{3 \cdot E \cdot I} = \frac{6.20 \cdot 6}{3 \cdot E \cdot I} = \frac{12.4}{E \cdot I}$$

$$\varphi_B = \frac{12.4 - 20.25}{E \cdot I} = \frac{-7.85}{E \cdot I} \text{ rad}$$

